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Inflation by The Einstein-Scalar-Gauss-Bonet Theory with Potential Inflation

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ABSTRACT

Research has been conducted on the inflation theory by Einstein-scalar-Gauss-Bonet theory with inflation potential to study the scenario of inflation. The purpose of this study is to calculate the inflation solution of the ESGB model. The research method used is a literature study with a mathematical approach. In this model, the Gauss-Bonet term is coupled with a scalar field so that it significantly modifies the dynamics of the early universe. The form of the scalar field used is, $f(\phi) = \lambda\phi^2$ and the inflation potential is quadratic, $V = \phi^2$. The lambda values used are $= 0{,}1; 0.2; 0.5; 1.; 2.0; 10$. For $\lambda \leq 0.5$, successfully demonstrate the inflationary solution, namely obtaining an exponentially expanding scale factor and a fixed value of the Hubble constant. In addition, the linear e-fold value is obtained by a linear graph and an exponentially decaying scalar field is obtained and an exponentially decaying scalar field is obtained. These results indicate that the ESGB model with inflationary potential can demonstrate the existence of an inflationary solution.

Keywords: Abstract, Inflation, Lambda, Gauss-Bonet

ABSTRAK

Telah dilakukan penelitian tentang teori inflasi oleh Eisntein-skalar-Gauss-Bonet teori dengan potensial inflasi untuk mengkaji skenario terjadinya inflasi. Tujuan dari penelitian ini adalah menghitung solusi inflasi dari model ESGB, menghitung spektral indeks dan rasio tensor skalar, serta membanding hasilnya dengan data observasi. Metode penelitian yang digunakan adalah studi literatur dengan pendekatan matematis. Dalam model ini, Suku Gauss-Bonet dikopelkan dengan medan skalar sehingga memodifikasi secara signifikan dinamika alam semesta awal. Bentuk medan skalar yang digunakan adalah, $f(\phi) = \lambda\phi^2$ dan potensial inflasi berbetuk kuadratik, $V = \phi^2$. Nilai lambda yang digunakan yang digunakan adalah $\lambda = 0{,}1; 0.2; 0.5; 1.; 2.0; 10$. Untuk $\lambda \leq 0.5$ berhasil menunjukan terjadinya inflasi, yaitu diperoleh faktor skala yang berekspansi secara eksponensial dan nilai konstanta *Huble* yang tetap. Selain itu, diperoleh nilai nilai e-fold yang linear dan medan skalar yang meluruh secara eksponensial. Hasil tersebut menunjukan bahwa model ESGB dengan potensial inflasi dapat menunjukan adanya solusi inflasi.

Kata kunci: Abstrak, Inflasi, Lambda, Gauss-Bonet

INTRODUCTION

General relativity is a fundamental theory in physics which successfully explains various cosmological phenomena, including the dynamics of the expansion of the universe and geometric structure of space-time. This theory has a limit to explain in the first universe, especially in explaining a number of major problems in modern cosmology, such as the horizon problem and the flatness problem (Dodelson, 2016). The horizon problem refers to the question of why the cosmic microwave background radiation (CMB) shows a very uniform temperature across the sky, even though in the standard model of the universe (based on general relativity), these regions should have never been causally related to each other since the beginning of time. Meanwhile, the flatness problem raises the question of why the curvature of space on cosmological scales is currently very close to zero, which requires a very fine-tuning of the initial conditions of the universe (Amendola & Tsijikawa, 2010).

To overcome these limitations, scientists have developed various modifications to the general theory of relativity. One of the most influential approaches is the concept of cosmological inflation, first proposed by Guth in the early 1980s. Inflation states that the universe underwent a very rapid exponential expansion in the early moments after the Big Bang (Guth, 1980). This inflationary period elegantly solves the horizon and flatness problems and provides a mechanism for the formation of primordial fluctuations that become the seeds for large structures in nature (Martin, 2004). However, the inflationary theory built on general relativity cannot explain the problem of the singularity at the beginning, namely the state where the energy density and curvature of spacetime become infinite (Arvind *et al.*, 2003). Therefore, the development of a new model built on Einstein's theory of gravity is needed to solve one of the singularity problems.

One suitable approach to avoid the singularity problem at the beginning of time is the Einstein-Scalar-Gauss-Bonnet (ESGB) theory, which adds a Gauss-Bonnet term coupled to the scalar field in the Einstein-Hilbert action (Kawai & Soda, 1999). The Gauss-Bonnet term arises naturally as a first-order correction in the effective action of string theory. The coupling of the scalar field (ϕ) and the coupling function $f(\phi)$ allows obtaining non-singular cosmological solutions that have a superinflation phase, namely exponential growth with increasing Hubble rate, (Kawai *et al.*, 1997; Soda *et al.*, 1998). In research by (Kanti *et al.*, 2015), the addition of the Gauss-Bonnet term to the Einstein-Hilbert action can produce solutions in the form of a scaling factor that experiences exponential growth.

However, the ESGB model also presents a new problem: it tends to be unstable against tensor perturbations without the inflation potential. These perturbations generate high-amplitude gravitational waves that disrupt the background geometry of spacetime. Adding the inflation potential $V(\phi) = m\phi^n$ to the ESGB model can produce a more stable model against tensor perturbations, particularly those with the form $0 < n < 5$ and whose coupling function is quadratic (Hikmawan *et al.*, 2017).

The ESGB model with inflation potential produces a spectrum of scalar and tensor disturbances that can be adjusted to the latest CMB observation data such as WMAP and plank data (Martin, 2004). This model can also explain the origin of primordial black holes and the formation of large structures with natural mechanisms from string theory (Kawai & Soda, 1999). Thus, based on the description above, we conducted research on Inflation by Einstein Scalar-Gauss-Bonnet Theory with Inflation Potential to obtain Inflation solutions from the ESGB model.

METHODOLOGY

Research in theoretical physics uses a mathematical approach by creating a cosmological model. The data sources used are secondary data in the form of journals and textbooks related to inflationary cosmology, general relativity, and modified gravity. Additionally, observational data, such as the 2018 Planck data, serve as a comparative reference. Observational data provide constraints or serve as a benchmark for the inflationary parameter, the spectral index n_s, and the tensor-scalar ratio (r). Figure 1 shows the research flow carried out in this study. The stages of this research are:

1. Literature study and problem identification: discussing the basic theory of cosmology
2. Theoretical model formulation: creating an Einstein-Gauss-Bonnet action model with additional scalar fields and inflationary potentials
3. Mathematical equation formulation: using the principle of variational action to obtain the equations of motion, namely the scalar field equations and the Friedman equations
4. Equation solution: The obtained differential equations are solved to obtain a solution. This can be done analytically, or if this is not possible, a numerical approach using Python is used.
5. Calculation of physical quantities: The obtained solutions are used to obtain inflation parameter values, namely scale factor (a(t)), e-folding value, N(t), Huble Constan, H_o, and scalar field,$\phi(t)$.
6. Discussion and interpretation: The research results are analyzed to examine physical implications, consistency with previous theories, and model parameter constraints.

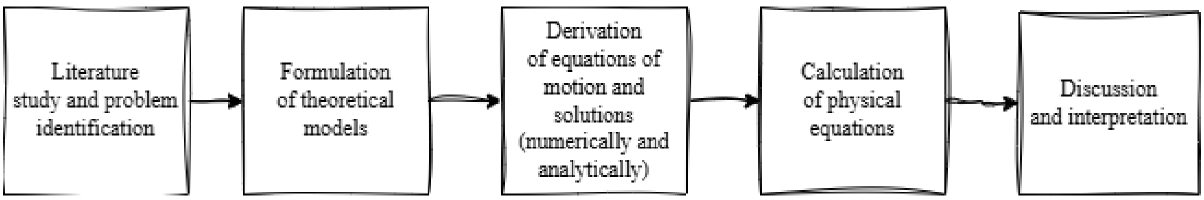

Figure 1. Flowchart of the research methodology
in the Einstein–Gauss–Bonnet inflationary model

RESULTS AND DISCUSSION

Einstein Scalar Gauss-Bonnet Field Equation

The reaction equation form of the Einstein action modification involving the Gauss-Bonnet (GB) theory form with inflation potential is (Kanti, 2015)

$$\mathcal{S} = \int d^4\sqrt{-g}\left[\frac{R}{2} - \frac{(\nabla\phi)^2}{2} + \frac{1}{8}f(\phi)R_{GB}^2 - V(\phi)\right] \tag{1}$$

Where the Gauss-Bonnet form R_{GB}^2 is defined as follows:

$$R_{GB}^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \tag{2}$$

Equation (1) is varied with respect to the scalar field and the metric $g^{\mu\nu}$. Equation (1) is varied with respect to the scalar field to obtain the following equation:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}\partial^\mu\phi\right] + \frac{1}{8}f'(\phi)R_{GB}^2 - V'(\phi) = 0 \tag{3}$$

with $f' = \frac{df}{d\phi}$

equation (1) is varied with respect to the electric $g^{\mu\nu}$:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + P_{\mu\alpha\nu\beta}\nabla^{\alpha\beta}f(\phi) = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}(\nabla\phi)^2 + V(\phi)\right] \tag{4}$$

with $P_{\mu\alpha\nu\beta}$ is defined as:

$$P_{\mu\alpha\nu\beta} = R_{\mu\alpha\nu\beta} + 2g_{\mu[\beta}R_{\nu]\alpha} + 2g_{\alpha[\nu}R_{\beta]\nu} + Rg_{\mu[\nu}g_{\beta]\alpha} \tag{5}$$

The metric used is the Friedmann-Lemaitre-Robertson-Walker (FRW) metric, the form of which is

$$ds^2 = -dt^2 + a^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \theta\, d\varphi)\right] \tag{6}$$

a(t) is the scale factor of the expansion of the universe and $k = 0 \pm 1$ which represents the tilt of the universe

The form of the universe used is a flat universe, namely the value of k = 0. so that equation (6) changes to,

$$ds^2 = -dt^2 + a^2[dr^2 + r^2(d\theta^2 + \theta\, d\varphi)] \tag{7}$$

All quantities in equations (3) and (4) are obtained from equation (7) so that we get the equation:

$$\ddot{\phi} + 3H\dot{\phi} - 3f'H^2(\dot{H} + H^2) + V'(\phi) = 0 \tag{8}$$

$$3H^3\dot{f} = \frac{\dot{\phi}^2}{2} + V(\phi) \tag{9}$$

$$2H(\dot{H} + H^2)\dot{f} + H^2\ddot{f} = -\frac{\dot{\phi}^2}{2} + V(\phi) \tag{10}$$

Equation (8) is subtracted from equation (9), then substituted into the equation $\dot{f} = f'\ \dot{\phi}$, and $\ddot{f} = (f''\ \dot{\phi} + f'\ \ddot{\phi})$ is obtained.

$$(H^3 - 2H\dot{H})f'\dot{\phi} - H^2(f''\dot{\phi}^2 + f'\ddot{\phi}) - \dot{\phi}^2) = 0 \tag{11}$$

Cases $f(\phi) = \lambda\phi^2$ and $V = \phi^2$

Equation (8) and (11) is simplified for the case of a quadratic coupling function and a scalar potential scalar $f(\phi) = \lambda\phi^2$, and $V = \phi^2$. Equations (7) and (10) change to:

$$\ddot{\phi} + 3H\dot{\phi} - 6\lambda H^2(\dot{H} + H^2) + 2\phi = 0 \tag{12}$$

$$2\lambda\left((H^3 - 2H\dot{H})\phi\dot{\phi} - H^2\dot{\phi}^2 + \phi\ddot{\phi})\right) - \dot{\phi}^2) = 0 \tag{13}$$

Equations (12) and (13) are second-order differential equations and can only be solved numerically. The initial parameters used are $\phi(0) = 1, \dot{\phi}(0) = 1, H(0) = 1$ with different variations in the value of λ. The results obtained from the numerical analysis are as shown in the figure 2.

The Einstein–Scalar–Gauss–Bonnet (ESGB) inflation model is a modified approach to the theory of general relativity that is able to explain the dynamics of the early universe more comprehensively, especially within the framework of cosmic inflation. In this model, the contribution of the Gauss–Bonnet coupling mediated by the scalar field ϕ is represented by the coupling parameter λ, which directly influences cosmological evolution. The presence of the GB term coupled to the inflationary potential is able to modify the dynamics of the early universe while providing predictions that are more consistent with the latest cosmological data (Odintsov *et al.*, 2020).

Inflation occurs when the scale factor, $a(t)$, experiences exponential expansion over time. Figure (1) shows a graph obtained from equations (11) and 12 solved numerically. The graph above shows the effect of the value of λ on the occurrence of inflation. $\lambda = 0.1$ (dark purple) and $\lambda = 0.2$ (dark blue) indicate that the scale factor experiences exponential expansion, although $\lambda = 0.1$ indicates that inflation occurs rapidly compared $to\ \lambda = 0.2$. The curve above also shows $\ddot{a} > 0$ which is marked by an upward concave curve. $\lambda = 0.5$ (light blue) indicates that the expansion rate is still increasing, although the graph is more sloping than the previous two. Inflation on this curve occurs more quickly than the previous ones, while other lambda values indicate that inflation does not occur as seen from the flat graph. The results above indicate that large values of λ inhibit cosmological expansion.

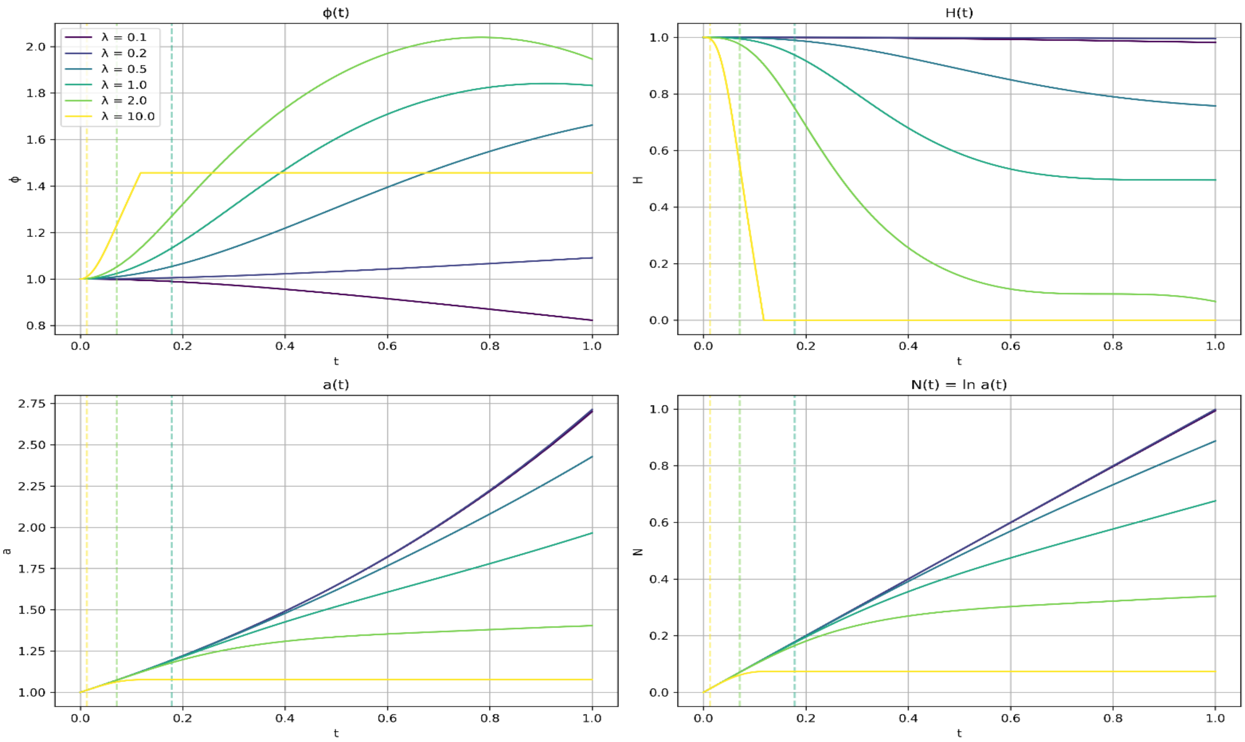

Figure 2. Graph of changes in the scale factor, $a(t)$, the expansion velocity field, $H(t)$, the e-fold value, $N(t)$, and the scalar field, (ϕ), against time with the value parameter $\lambda = 0.1; 0.2; 0.5; 1.0; 2.0; 10.0$

In addition to the scale factor, the value of the Huble constant (H_0) is an important parameter because it can determine the current rate of expansion of the universe. The current value of H_0 based on the data of the Microwave Background Radiation (CMB) anisotropy analysis using the ΛCDM model is

$$H_0 = 67{,}4 \pm 0.5 \frac{km}{s}/Mps \tag{13}$$

(Planck Collaboration *et al.*, 2016)

The graph in figure one shows lambda values of 0.1 *and* 0.2, indicating a constant Hubble constant. This graph shows an expansion approaching de-sitter expansion, namely successful and stable inflation. The blue-green graph (λ = 0.5; 1.0; 2.0) shows inflation occurring but experiencing a slowdown, while the yellow graph shows no inflation. For small λ indicates inflation where the H value increases then decreases and remains constant, while large λ values indicate a decreasing H value and approaches zero.

The evolution graph $N(t) = ln\, ln\, a(t)$ provides direct information about the total amount of exponential expansion of the universe. For small values of λ, the $N(t)$ curve increases almost linearly with time, indicating that the scale factor $a(t)$ grows exponentially. Thus, the amount of e-folding reaches significant values, which are theoretically necessary to solve early problems of cosmology such as the horizon and flatness problems (Martin, 2008).

However, for larger values of λ, the $N(t)$ The curve saturates early, even stagnating at low values. This indicates that the total amount of e-folding produced is too small. In the context of inflationary cosmology, this condition means the model is unable to provide a sufficiently long inflation period, and therefore cannot explain the homogeneity of the cosmic background radiation (CMB) or the smoothness of the observed large-scale structure.

These findings indicate a limitation on the value of the λ parameter in EGB inflation models. Too large a λ value not only inhibits the initial expansion rate but also limits the amount of e-folding, a key indicator of the success of an inflation scenario. Therefore, the parameter needs to be carefully calibrated so that the model can produce inflation that is sufficiently long and consistent with Planck 2018 and BICEP/Keck Array observations (Akrami *et al.*, 2020)

The $\phi(t)$ graph shows that for small values of λ, the scalar field undergoes dynamic evolution and increases significantly, reaching a peak before gradually decreasing, signaling the end of inflation. In contrast, for $\lambda = 10.0$ the evolution of $\phi(t)$ is very slow and tends to be static, indicating that the field is not sufficiently driven to create efficient inflationary conditions. This is consistent with the hypothesis that increasing the contribution of the Gauss-Bonnet term causes a slowdown in the dynamics of $\phi(t)$, thus reducing the resulting inflationary potential. The Gauss-Bonnet effect can be very influential when passing the slow-roll approximation. The quadratic form of the slow-roll approximation potential is still valid for predicting n_s and r values, but is invalid for the quartic form. This shows that the Gauss-Bonnet term is not only a small correction but can produce spectral index predictions that agree well with observational observations (Mudrunka & Nakayama, 2025).

According to (Solbi & Karami, 2024), their research found that inflation with a Gauss-Bonnet term and non-minimal coupling can produce increased curvature fluctuations on small scales. This mechanism leads to the possibility of the formation of primordial black holes with masses around $10^{-14} M_{\odot}$, which are prime candidates for dark matter. Furthermore, the ESGB model can also produce gravitational wave backgrounds from the induced scalar model. In an extension of the model,(Koh *et al.*, 2024) conducted a study of Higgs-based inflation with an additional Gauss-Bonnet term, showing that the Gauss-Bonnet term becomes dominant only during the inflationary era and subsides after the inflationary era ends, therefore, it does not contradict the constraints on gravitational wave propagation.

In general, the results of research on the ESGB theory with an inflationary potential not only improve the standard model but also open up possibilities for explaining standard cosmological phenomena. The ESGB model is also consistent with measurements of the CMB and can explain the origin of dark matter and predict gravitational waves.

CONCLUSION

This study shows that the Einstein–Scalar–Gauss–Bonnet (ESGB) inflation model with an inflationary potential provides a consistent solution to explain the early phase of the universe. By incorporating a Gauss–Bonnet term quadratically coupled to the inflaton field, it is found that the value of the parameter λ is crucial for the success of the inflationary scenario. For small λ, the scale factor undergoes exponential expansion, and the amount of e-folding is quite large. However, for too large λ, the inflation process is hampered and unable to produce a sufficient amount of e-folding. This finding emphasizes the importance of parameter calibration in the ESGB model to ensure consistency with observational data. Overall, the ESGB model with an inflationary potential not only extends the standard inflationary model but also provides a richer picture of cosmological phenomena, including the formation of primordial black holes and the gravitational wave background.

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