E-ISSN-2580-7129
Print- ISSN-1978-6107
TECHNO: JURNAL PENELITIAN
Journal homepage: http://ejournal.unkhair.ac.id/index.php/Techno
Volume 15 Number 01 May 2026 DOI: http://dx.doi.org/10.

Response Surface–Based Multi-Objective Optimization of Energy Absorption and Reaction Force in Axially Impacted Thin-Walled Aluminum Alloy Tube

Witono Hardi[1]*, Ambar Sulistyo Wardani[2], Mohammad Shalahuddin Abdul Aziz[3]

[1]Program Studi Teknik Mesin, Universitas Khairun, Jalan Jusuf Abdul Rahman, Ternate Selatan, Kota Ternate, Maluku Utara, Indonesia, email witono@unkhair.ac.id
[2]Program Magister Manajemen, Konsentrasi Manajemen Sistem Informasi, Universitas Merdeka Malang, Jalan Pisang Candi, Kota Malang, Jawa Timur, Indonesia
[3]Master of Professional Engineering (Mechatronic), Faculty of Sciences, Engineering and Technology, Adelaide University, North Terrace, Adelaide SA 5005, Australia

Received : 27-01-2026
Accepted : 11-05-2026
Available online : 31-05-2026

ABSTRACT

Thin-walled aluminum alloy tubes are widely used as energy-absorbing elements in safety structures because they exhibit strong plastic deformation under axial impact loads. This study aims to optimize the axial impact response of aluminum alloy tubes by simultaneously maximizing absorbed impact energy and minimizing the maximum reaction force. The analysis was carried out using numerical simulation of the element method, with a fixed tube length of 50 mm and a constant impact velocity. Varied geometric parameters include tube diameter (15–20 mm) and wall thickness (0.5–1.5 mm). The experimental design was prepared using Central Composite Design (CCD) within the framework of the Response Surface Methodology (RSM). The RSM quadratic model was developed to map the relationships between geometric parameters and the responses of absorbed energy and maximum reaction force. The simulation results show a clear trade-off between the two responses: an increase in absorbed energy is accompanied by an increase in the reaction force. Multi-response optimization is performed using a desirability function approach with equal weights for both responses. The optimization results showed that the optimal design was achieved at a diameter of about 19 mm and a thickness of about 1.1 mm, yielding high absorbed energy with relatively low reaction forces. The methodology and results of this study provide a systematic basis for the design of energy-absorbing elements using aluminum alloy tubes for crashworthiness applications.

Keywords: axial impact; aluminum tubes; energy absorption; multi-response optimization; RSM

ABSTRAK

Tabung berdinding tipis dari paduan aluminium banyak digunakan sebagai elemen penyerap energi pada struktur keselamatan karena kemampuan deformasi plastisnya yang baik di bawah beban tumbukan aksial. Penelitian ini bertujuan untuk mengoptimalkan respon tumbukan aksial tabung paduan aluminium dengan memaksimalkan energi impak yang terserap dan meminimalkan gaya reaksi maksimum secara simultan. Analisis dilakukan menggunakan simulasi numerik metode elemen hingga dengan panjang tabung tetap 50 mm dan kecepatan tumbukan konstan. Parameter geometri yang divariasikan meliputi diameter tabung (15–20 mm) dan ketebalan dinding (0,5–1,5 mm). Desain eksperimen disusun menggunakan Central Composite Design (CCD) dalam kerangka Response Surface Methodology (RSM). Model kuadratik RSM dikembangkan untuk memetakan hubungan antara parameter geometri dan respon energi terserap serta gaya reaksi maksimum. Hasil simulasi menunjukkan adanya trade-off yang jelas antara kedua respon, di mana peningkatan energi terserap diikuti oleh kenaikan gaya reaksi. Optimasi multi-respon dilakukan menggunakan pendekatan desirability function dengan bobot yang seimbang untuk kedua respon. Hasil optimasi menunjukkan bahwa desain optimum diperoleh pada diameter sekitar 19 mm dan ketebalan sekitar 1,1 mm, yang menghasilkan energi terserap tinggi dengan gaya reaksi yang relatif rendah. Metodologi dan hasil penelitian ini memberikan dasar yang sistematis untuk perancangan elemen penyerap energi berbasis tabung paduan aluminium pada aplikasi crashworthiness.

Kata Kunci: tumbukan aksial; tabung aluminium; penyerapan energi; optimasi multi-respon; RSM

INTRODUCTION

The need for lightweight structures that can effectively absorb impact energy is increasing as transportation technology and structural safety demands develop (Witteman, 1999; Alghamdi, 2001; Ahmad, 2009; Hardi;2022; Ferdynus at.al, 2020). Crashworthiness in the energy-absorbing structural system is very important because it determines a component or vehicle's ability to protect passengers in the event of a collision by effectively absorbing and distributing collision energy. Without good crashworthiness, the impact energy will be directly transmitted to the human body, increasing the risk of fatal injury. With a crashworthy design, the structure can undergo controlled deformation, maintain the integrity of the passenger compartment, and reduce the acceleration and forces experienced by humans. It is not only related to life safety, but it is also becoming a major standard in automotive engineering, transportation, and safety-related industrial equipment (Zarei and Kröger, 2007; Xu et al, 2022; Mathiazhagan et al, 2023).

Aluminum alloy is widely used for energy-absorbing elements because it offers a high strength-to-mass ratio, good corrosion resistance, and stable plastic deformation under axial impact loads. In axial impact conditions, aluminum alloy tubes generally undergo progressive folding, which significantly contributes to the energy-absorption mechanism. (Galib and Limam, 2004). Nevertheless, the impact response characteristics of the tube are determined not only by the material but also by its geometric parameters, particularly the diameter and wall thickness (Ghamarian and Abadi, 2011; Idesman and Mates, 2014).

Previous studies have shown that increased energy absorption capacity is often accompanied by increased maximum reaction force, which can negatively impact the safety of the structure or the user. Therefore, the design of energy-absorbing elements requires an optimization approach that does not focus on a single response but considers several simultaneous, conflicting responses. Conventional trial-and-error approaches or variations of one parameter at a time become less efficient and fail to capture the complex interactions among design parameters. Toksoy and Güden, 2011; Aktaş et al, 2022).

The Response Surface Methodology (RSM) offers a systematic statistical approach to model the nonlinear relationship between design parameters and structural responses, and allows for efficient multi-response optimization. By combining numerical impact simulation with RSM, the design space can be explored comprehensively with a relatively small number of simulations. However, applying multi-response RSM to optimize the axial impact behavior of aluminum alloy tubes, particularly by simultaneously considering absorbed energy and maximum reaction force, still requires further in-depth study to develop applicable design guidelines based on quantitative analysis.

The axial impact behavior of thin-walled aluminum alloy tubes is a nonlinear mechanical phenomenon governed by complex interactions among material properties, dynamic loading conditions, and geometric structural parameters. (Jones, 1989) . In particular, variations in the tube wall diameter and thickness directly affect the progressive plastic deformation mechanism, energy absorption capacity, and the magnitude of the maximum reaction force generated during the impact process. However, an increase in absorbed energy is generally accompanied by an increase in structural rigidity, which, in turn, increases the reaction force, creating an intrinsic conflict between design responses (Lu and Yu, 2003). Therefore, a modeling framework is needed that can quantitatively represent the nonlinear relationships and interactions among geometric parameters while accommodating multi-response optimization in the axial impact system of aluminum alloy tubes.

This study aims to develop a quadratic model based on Response Surface Methodology (RSM) to characterize the influence of the diameter and wall thickness of aluminum alloy tubes on the absorbed impact energy and the maximum reaction force under axial impact conditions. In addition, this study aims to apply multi-response optimization using a statistical approach to obtain a geometric configuration that is theoretically optimal in maximizing energy absorption capacity while minimizing maximum reaction force. The results of the study are expected to enrich the fundamental understanding of the mechanical trade-offs in thin-walled tube structures and to make a conceptual contribution to the development of crashworthiness optimization methodologies based on numerical simulations.

METHODOLOGY

In general, this research can be described as figure 1:

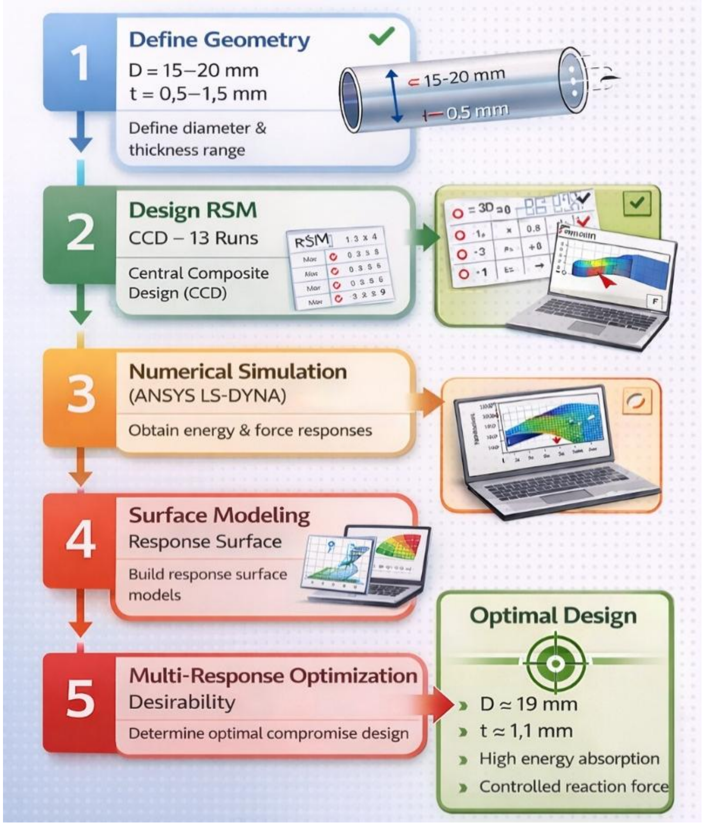

Figure 1. Research Flow

This research used a numerical simulation approach based on the element method to analyze the axial impact behavior of thin-walled aluminum alloy tubes. A numerical approach was chosen to allow detailed observation of the plastic deformation mechanism and force response during the impact process with a high degree of parameter control. All simulations were carried out under a constant maintained impact velocity to isolate the influence of geometric parameters on structural response. Some crashboxes also use numerical approaches in research solutions (Günaydın et al, 2023).

As shown in Figure 1, the study's object is an aluminum alloy tube with a fixed length of 50 mm, with the diameter and wall thickness as the main design parameters. The diameter range is 15 to 20 mm, while the wall thickness varies between 0.5 and 1.5 mm. The selection of this parameter range is based on mechanical considerations to keep the tube within the progressive plastic deformation domain relevant to energy-absorption applications, while avoiding unrepresentative structural failures.

The material used in this study is an aluminum alloy, which is modeled as an isotropic elastoplastic material. The selection of aluminum alloys is based on their mechanical characteristics, which are suitable for energy-absorbing applications, in particular their high strength-to-mass ratio and stable plastic deformation under dynamic loading. The material properties used in the simulation are taken from the literature on the axial impact study of aluminum tubes and summarized in Table 1.

Table 1. Material Properties

Material Properties	**Simbol**	**Value**
Density	ρ	2700 kg/m³
Modulus of elasticity	E	70 GPa
Poisson Ratio	v	0,33
Yield	σy	250 MPa
Ultimate Tensile Strength	σu	290 MPa
Fracture strain	εf	0,12

This material model is assumed to adequately represent the global plastic behavior of aluminum alloys under the analyzed axial impact conditions. The effect of strain rate was not explicitly considered in this study, as the focus was on geometry optimization and interactions among design parameters.

Figure 2 illustrates a finite element analysis (FEA) workflow for a cylindrical component attached to a rigid block, shown in four stages:

Modeling (top-left)
A 3D geometric model is created. It consists of a hollow or solid cylindrical tube rigidly connected to a rectangular block, representing the physical structure before numerical analysis.

Meshing (top-right)
The geometry is discretized into finite elements. The cylinder is meshed with fine, structured elements along its length and circumference, while the block is meshed with a coarser, regular grid. This step prepares the model for numerical computation.

Solution (bottom-left)
The analysis results are visualized as a color contour plot on the deformed shape. The color gradients (blue to green/yellow) represent the distribution of a field variable—typically stress, strain, or displacement—indicating higher and lower response regions in the cylinder, especially near the interface with the block.

Slice View (bottom-right)
A longitudinal cross-sectional (cut) view of the cylinder is shown. This reveals the internal distribution of the solution variable, allowing inspection of gradients and concentrations inside the material that are not visible on the outer surface.

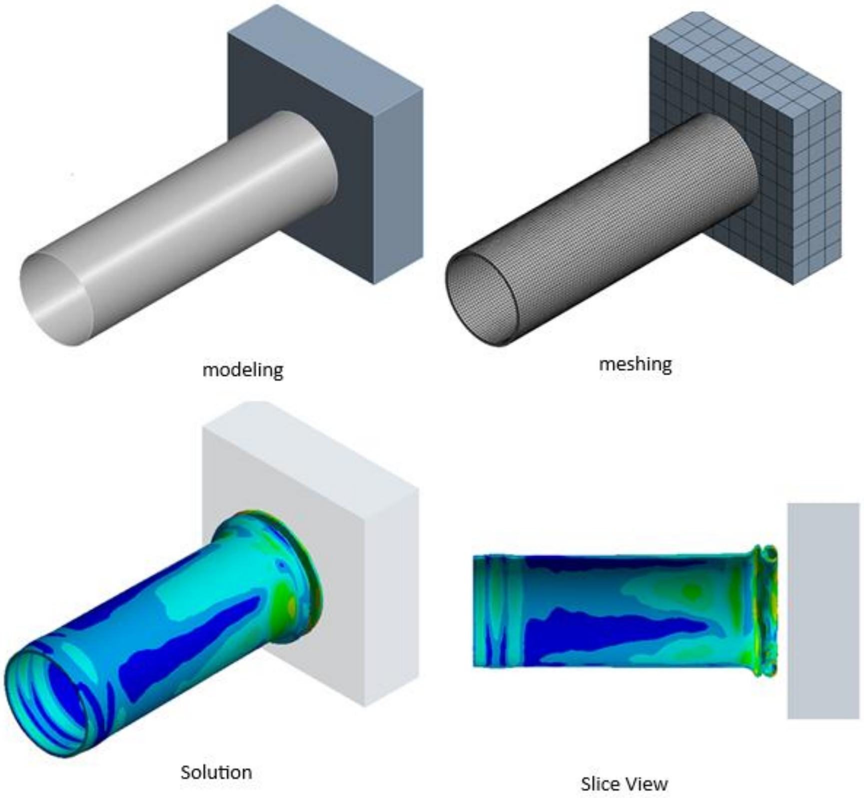

Figure 2. Simulation Processes

The numerical experiment design was prepared using Central Composite Design (CCD) within the framework of the Response Surface Methodology (RSM). CCDs were chosen for their ability to construct effective quadratic models to represent the nonlinear relationships between design variables and system responses with a relatively efficient number of simulations. The two main responses analyzed are absorbed impact energy and maximum reaction force, which represent the energy-absorption capacity and the dynamic stiffness of the structure, respectively.

Each CCD design configuration is analyzed using an axial impact simulation, in which one end of the tube is held, and the other is subjected to an axial impact. The nonlinear contact between the fist and the tube is modeled to capture dynamic interactions realistically. In contrast, the tube's plastic deformation is represented using an elastoplastic material model suitable for aluminum alloys. During the simulation, the internal reaction force and energy data are recorded as a function of time, then used to determine the maximum response value that is further analyzed.

To build a Response Surface Methodology (RSM) model that represents the nonlinear relationship between geometric parameters and impact response, a Central Composite Design (CCD) is used as the experimental design. CCDs were chosen for their efficiency in producing quadratic models with a relatively small number of simulations compared to the full experimental approach.

This study involved two design factors: the tube diameter (D) and the tube wall thickness (t). By the number factor, the total number of CCD runs is determined based on the following combination of design components$k = 2$:

Factorial points are as many as the runs, representing the extreme combinations of the low and high levels of each factor.$2^k = 4$

Axial points are as many as the runs, used to capture curvature and strengthen the capabilities of quadratic models.$2k = 4$

The center point has 5 runs, which are used to evaluate the simulation response's stability and ensure numerical consistency.

Thus, the total number of numerical simulation runs used in this study is:

$$N = 2^k + 2k + n_c = 4 + 4 + 5 = 13 \text{ run}$$

The use of multiple central points provides a solid basis for evaluating the reliability of the RSM model in deterministic simulations, thereby increasing confidence in the optimization results. The number of runs is considered sufficient to build a representative two-variable quadratic model without excessively increasing the computational load.

Based on simulation data, a quadratic RSM model was developed for each response using regression analysis. This model includes linear and quadratic effects, as well as interactions between the tube wall diameter and thickness. The validity of the model is evaluated through variance analysis (ANOVA) to ensure that the model obtained can represent the relationship between design variables and responses in a statistically significant manner.

The optimization stage is carried out using a multi-response approach based on the desirability function, with the absorbed energy as the maximized response and the maximum reaction force as the minimized response. Both responses are given balanced weight to obtain a compromise solution that reflects the balance between energy absorption capability and reaction force limitation. The highest desirability value in the analyzed design domain determines the optimal solution. For the response you want to maximize (energy absorbed):

$$d_E = \begin{cases} 0, & E < E_{\min} \\ \left(\dfrac{E - E_{\min}}{E_{\max} - E_{\min}}\right)^s, & E_{\min} \le E \le E_{\max} \\ 1, & E > E_{\max} \end{cases}$$

For the response you want **to minimize** (maximum reaction force):

$$d_F = \begin{cases} 1, & F < F_{\min} \\ \left(\dfrac{F_{\max} - F}{F_{\max} - F_{\min}}\right)^t, & F_{\min} \le F \le F_{\max} \\ 0, & F > F_{\max} \end{cases}$$

where:
$d_E$dan d_F is individual desirability,
$E_{\min}, E_{\max}$ is the minimum and maximum energy limit,
$F_{\min}, F_{\max}$ is the minimum and maximum force limit,
s and t is the weight parameter (taken the same for balanced optimization).

As a verification step, the RSM model's predictions are compared with direct numerical simulation results to assess the model's accuracy. The alignment between the prediction and simulation results is used as an indicator of the reliability of the RSM approach in representing the axial impact behavior of aluminum alloy tubes. Thus, the methodology applied is not only descriptive but also provides a systematic analytical framework for optimizing energy-absorbing structures through numerical simulations.

RESULTS AND DISCUSSION

The results of the Axial Impact Simulation are the absorbed energy and the reaction force. Two response variables, namely absorbed energy and reaction force, are shown in Figure 3.

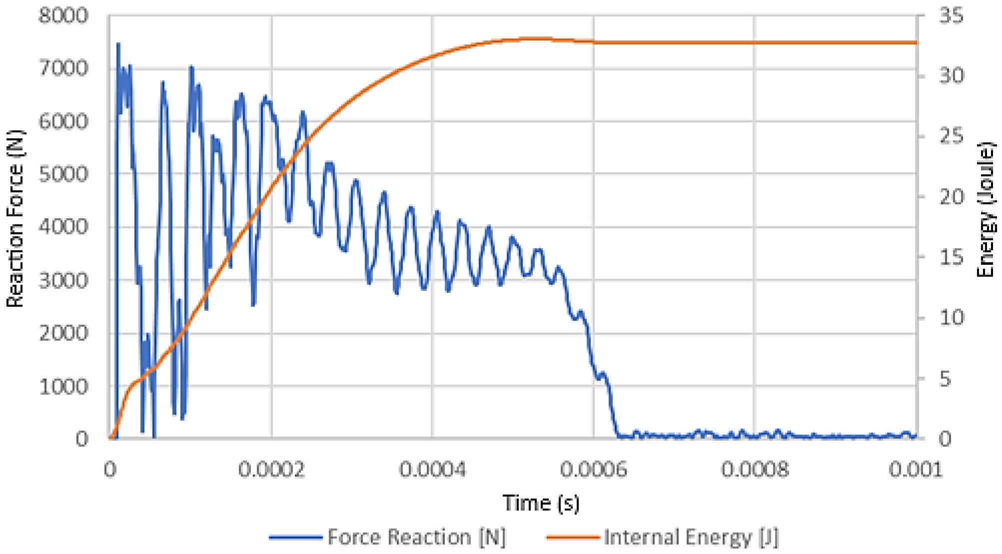

Figure 3. Reaction Force and Absorbed Energy

Numerical simulation of axial impact was performed on aluminum alloy tubes with a fixed length of 50 mm, constant impact speed, and variations in diameter and thickness according to the Central Composite Design (CCD) design. The two main response parameters analyzed were the absorbed impact energy (E) and the maximum reaction force (F).

The simulation results in Table 2 show that the absorbed energy ranges from 28,429 to 39,847, while the maximum reaction force ranges from 5,059 to 30,378. The response values at the design center point (diameter 17.5 mm and thickness 1.0 mm) were identical across simulation iterations, indicating that the simulation was deterministic and numerically stable. In general, increasing the tube's diameter and thickness increases absorbed energy, but it also increases the maximum reaction force. This confirms the fundamental trade-off between the ability to absorb energy and the magnitude of the peak force transmitted to the supporting structure.

Table 2. Simulated results

Run	Diameter (mm)	Thickness (mm)	Energy Absorbed (Joule)	Maximum Reaction Force (N)
1	15.0	0.5	33 055	7 462.7
2	20.0	0.5	32 879	10 030
3	15.0	1.5	31 691	22 466
4	20.0	1.5	39 847	30 378
5	13.96	1.0	35 859	13 790
6	21.04	1.0	37 376	21 401
7	17.5	0.29	28 429	5 059.5
8	17.5	1.71	31 825	29 943
9	17.5	1.0	36 407	17 368
10	17.5	1.0	36 407	17 368
11	17.5	1.0	36 407	17 368
12	17.5	1.0	36 407	17 368
13	17.5	1.0	36 407	17 368

Model Response Surface Methodology (RSM)

Based on the simulation data, an RSM quadratic model was built for each response using least squares regression. Common models used are:

$$Y = \beta_0 + \beta_1 D + \beta_2 t + \beta_{12} D t + \beta_{11} D^2 + \beta_{22} t^2$$

where D is the diameter of the tube, and is the thickness of the wall.t

The RSM model can represent the nonlinear relationship between geometric parameters and impact response well, particularly in capturing interactions and curvatures that simple linear models cannot explain. Consistent response values at a central point strengthen the model's reliability in predicting system behavior.

Effect of Geometric Parameters on Absorbed Energy

The results of the response surface and contour plot analysis showed that the absorbed energy increased with increasing tube diameter. The larger diameter increases the cross-sectional moment of inertia, allowing greater plastic deformation during axial impact.

The wall thickness also has a significant effect on the absorbed energy, but with nonlinear characteristics. At too small a thickness, the tube fails prematurely due to local buckling or unstable folding, resulting in low absorbable energy. Conversely, at too large a thickness, the structure becomes too rigid, reducing the plastic deformation mechanism and decreasing the energy absorption efficiency.

This phenomenon aligns with the findings of Sun et al. and Baroutaji et al., who reported an optimal thickness that maximizes absorbed energy in a thin-walled aluminum tube under axial impact.

Influence of Geometric Parameters on Maximum Reaction Force

In contrast to absorbed energy, the maximum reaction force increases almost monotonically with tube diameter and thickness. Wall thickness is the dominant parameter in increasing the peak force, as greater thickness directly increases the structure's axial rigidity. The contour plot of the force shows that the combination of a large diameter and a large thickness yields the highest maximum reaction force. This condition is undesirable in crashworthiness applications, as high peak forces can cause serious damage to support structures or increase the risk of passenger injury. These findings are consistent with the basic concept of impact mechanics, in which overly rigid structures tend to transmit greater forces rather than absorb energy through progressive plastic deformation.

Energy and Force Trade-Off Analysis

The simulation results clearly show a conflict between two design objectives:

Design with maximum absorbed energy (large diameter and thickness) is always associated with a high maximum reaction force. In contrast, designs with minimum force are achieved on very thin-walled tubes, but at the expense of inadequate energy absorption.

Pareto plots and desirability space show that the optimal solution is not at the extreme limits of the design space, but rather in the region of compromise. This condition confirms the importance of a multi-response optimization approach in the design of energy-absorbing structures.

Multi-Response Optimization Using the Desirability Function

Multi-response optimization is carried out using a desirability function approach, with the following criteria: Absorbed energy is maximized and Maximum reaction force minimized.

The optimization results show that the most balanced combination of geometric parameters is obtained at: Diameter approx. 19 mm; Thickness approx. 1.1 mm

In this configuration, the absorbed energy remains high (more than 90% of the maximum value), while the maximum reaction force can be significantly lower than in extreme designs. This condition represents the optimal design compromise for crashworthiness applications.

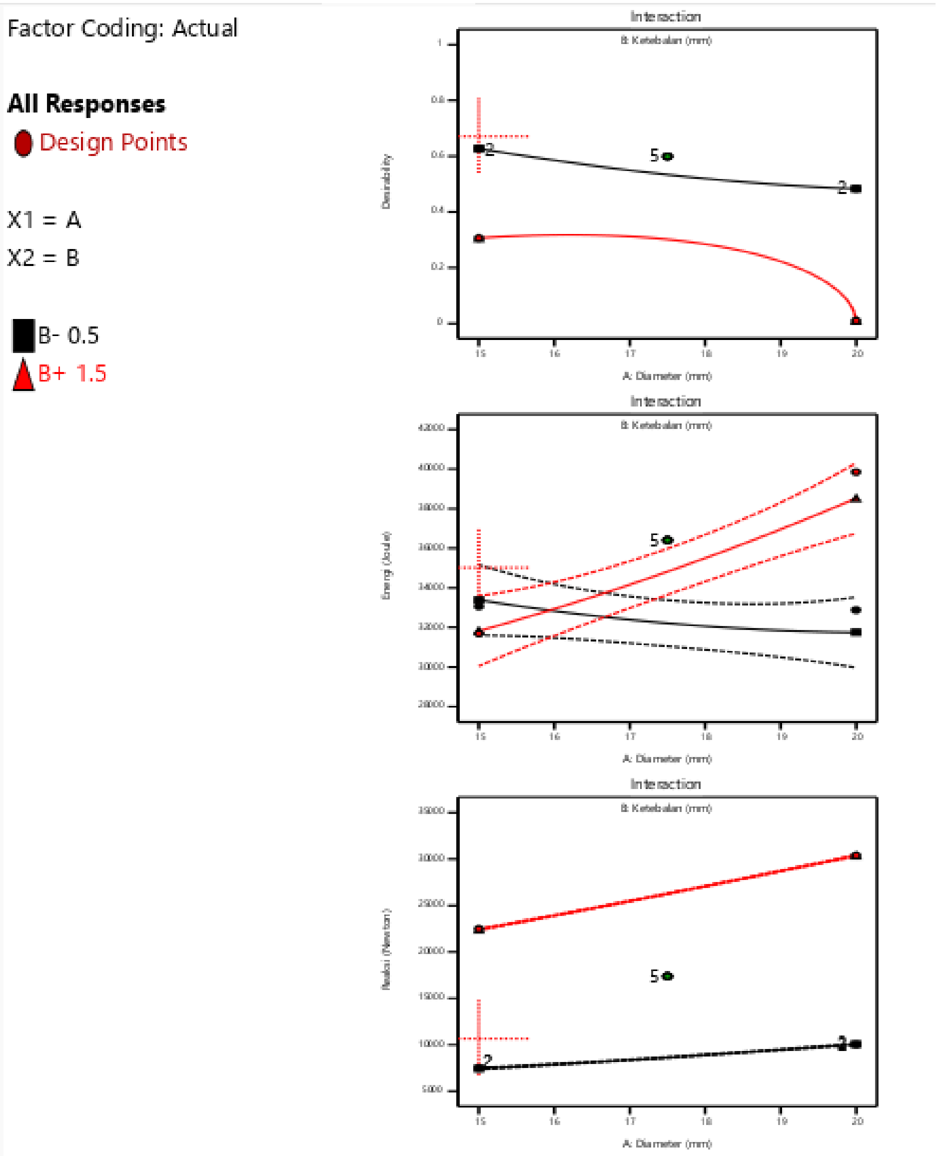

Figure 4. *Plot of Response Surface Methodology (RSM) interaction between tube diameter (A) and wall thickness (B) on the functions of desirability, absorbed energy, and maximum reaction force at axial impact of aluminum alloy tubes.*

Figure 4 shows the plot of the RSM interaction between tube diameter (A) and wall thickness (B) against three main responses, namely desirability, absorbed energy, and maximum reaction force, with factors encoded in actual values (*factor coding: actual*). The plot of the desirability function (top) shows that the desirability value does not increase monotonically with increasing tube diameter, especially at high wall thickness. In low-thickness conditions (B = 0.5 mm), an increase in diameter only provides a limited increase in desirability. In contrast, under high-thickness conditions (B = 1.5 mm), desirability decreases with increasing diameter. This indicates that extreme design does not produce an optimal multi-response solution and confirms the *trade-off* between absorbed energy and reaction force.

The absorbed energy plot (center) shows that the impact energy increases significantly with increasing tube diameter at a high wall thickness (B = 1.5 mm). In contrast, at a low wall thickness, the trend is relatively steeper. This difference in curve slope indicates a strong interaction between diameter and thickness, where the influence of diameter on absorbed energy becomes more dominant in thicker structures.

The plot of the maximum reaction force (bottom) shows that it increases almost linearly with tube diameter for both thickness conditions. However, the reaction force at high thickness is consistently much greater than at low thickness, which indicates that wall thickness is the dominant factor in increasing the axial rigidity of the structure. This tendency explains why an increase in energy absorption in a thick-walled design is always followed by an increase in the maximum reaction force.

These three interaction plots confirm that optimizing the axial impact behavior of aluminum alloy tubes cannot be achieved by maximizing or minimizing a single response. The highest desirability values are obtained in the medium-diameter and moderate-thickness regions, which represent an optimal compromise between increased absorbed energy and maximum control of the reaction force.

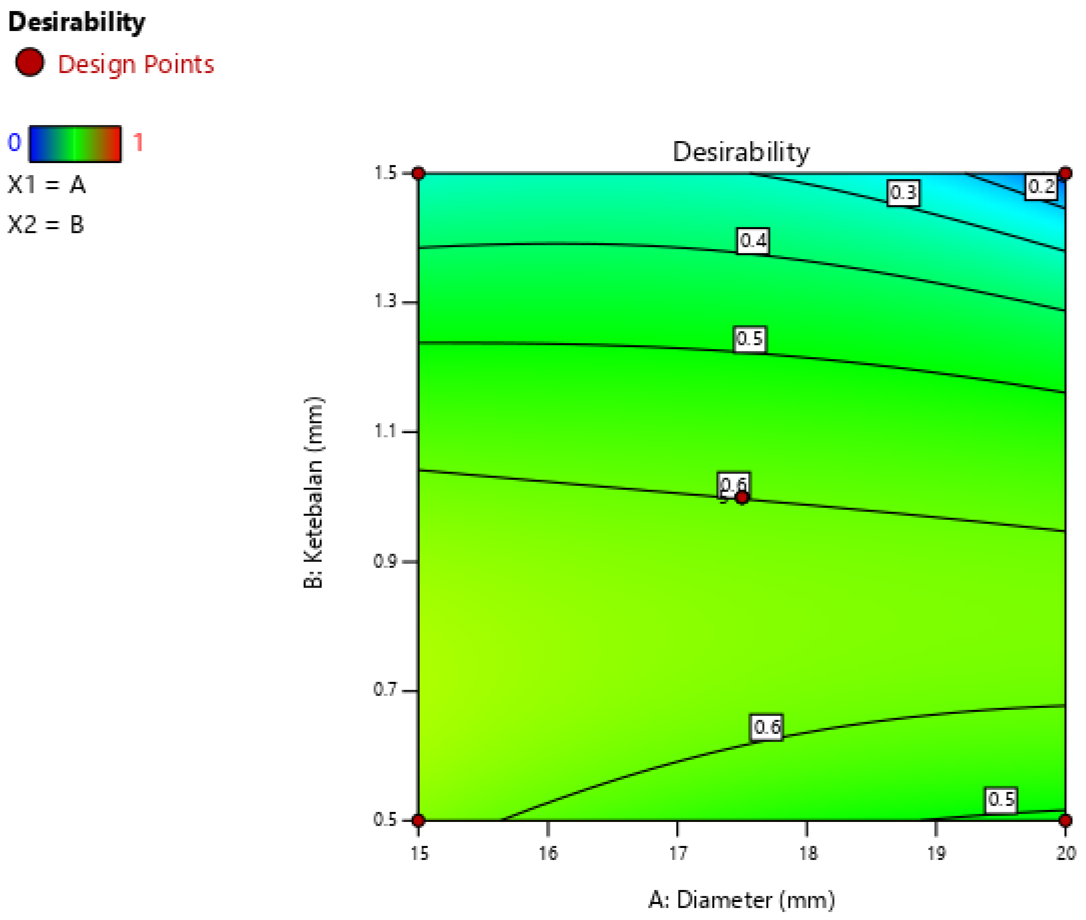

Figure 5. *Contour plot of the desirability function results of multi-response RSM optimization showing the influence of tube diameter (A) and wall thickness (B) on desirability values on axial impact of aluminum alloy tubes.*

Figure 5 shows that the highest desirability value is not at the extreme limits of the design domain, but rather in the medium-to-large-diameter and moderate-thickness regions. The decrease in desirability at large thicknesses is due to an increase in the maximum reaction force, whereas at small thicknesses it is due to low absorbed energy. The design points with the highest desirability values are in the range of about 17.5–19 mm in diameter and 1.0–1.2 mm in thickness, representing a compromise between energy absorption and reaction force control. The presence of design points in this area indicates consistency between the numerical simulation results and the RSM model predictions. These results confirm that the optimal configuration is achieved through a compromise between increased absorbed energy and control of the maximum reaction force.

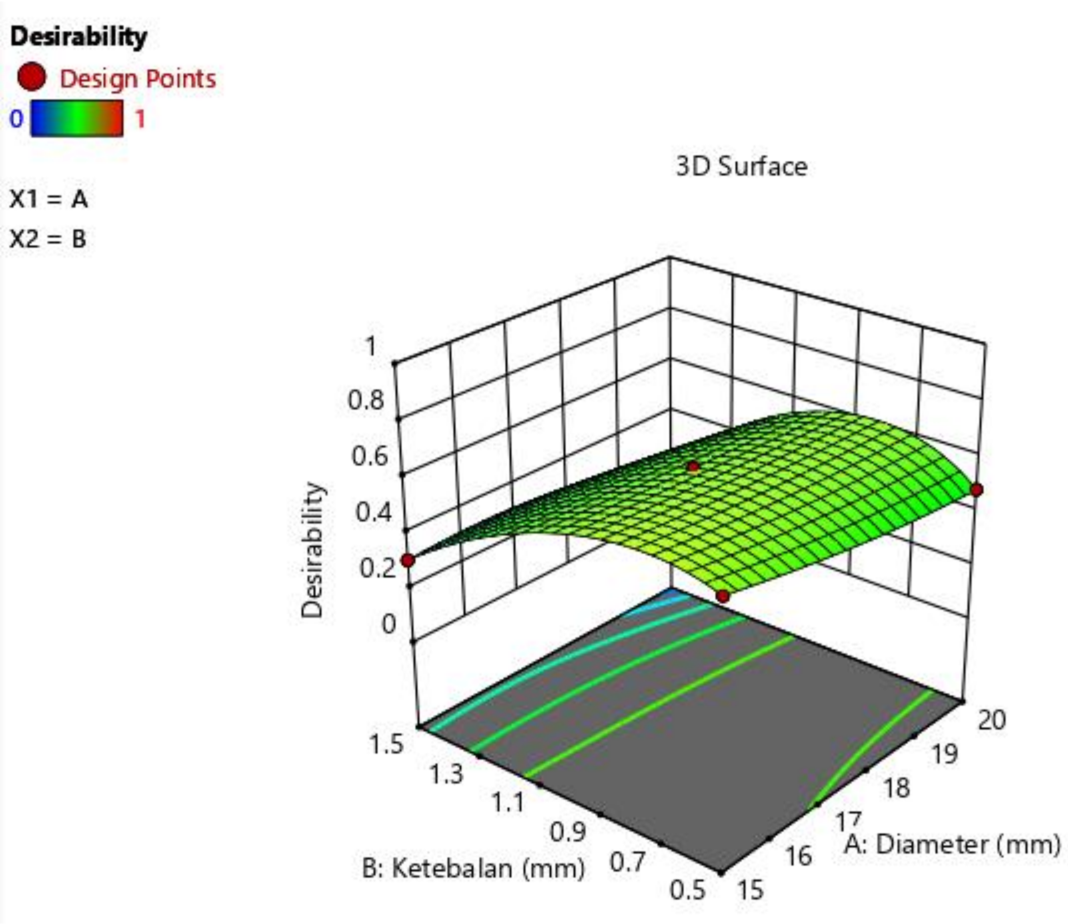

Figure 6. a three-dimensional surface plot (3D surface)

Figure 6 shows a three-dimensional (3D surface) plot of the desirability function resulting from the multi-response RSM optimization as a function of tube diameter (A) and wall thickness (B). The surface desirability shows that the desirability value increases with diameter, reaching the medium-high range, peaks at moderate wall thickness, and then decreases again at greater thickness.

The peak zone of the desirability surface is approximately 17.5–19 mm in diameter and 1.0–1.2 mm in thickness, representing a compromise between increased absorbed energy and maximum reaction force control. The decrease in desirability at large thicknesses reflects the predominance of an increase in reaction forces, whereas a decrease at small thicknesses is due to low energy absorption. The design points displayed on the prediction surface show consistency between the simulation data and the RSM model.

Engineering Implications

The results of this study show that optimizing the geometry of aluminum alloy tubes cannot be based on a single criterion. Designs that pursue maximum energy alone can generate harmful reaction forces, while designs that suppress forces alone will fail to fulfill the function of energy absorption.

The multi-response RSM approach applied in this study provides an efficient and systematic framework for identifying optimal design by considering both aspects simultaneously. This methodology is relevant for the development of energy-absorbing components in vehicle structures, industrial protection systems, and other lightweight structure applications.

CONCLUSION

Based on numerical simulations of axial impact of aluminum alloy tubes and analysis using the Response Surface Methodology (RSM), it can be concluded that variations in tube diameter and wall thickness significantly influence the absorbed impact energy and the maximum reaction force. The absorbed energy from the entire design configuration is 28,429–39,847 N, while the maximum reaction force is 5,059.5–30,378 N. Increasing diameter and thickness generally increases absorbed energy but also increases reaction forces due to the structure's increased axial rigidity.

The RSM quadratic model successfully represents the nonlinear relationships and interactions between the geometric parameters of the tube and the two responses. Wall thickness and diameter–thickness interactions were identified as the dominant factors influencing impact behavior. This is reflected in the contrast in response between the ultra-thin-walled configuration (0.29 mm thickness), which produces a minimum absorbed energy of 28,429 with a minimum reaction force of 5,059.5 N, and the thick-walled configuration (1.5 mm thickness and 20 mm diameter), which produces a maximum absorbed energy of 39,847 joule but with a very high maximum reaction force of 30,378 N.

The results of multi-response optimization using the desirability function approach indicated that the optimal design compromise was achieved at a diameter of about 19 mm and a thickness of about 1.1 mm. In this configuration, the predicted absorbed energy is in the range of 36,800–37,500 joule, or more than 90% of the observed maximum energy, while the maximum reaction force can be suppressed in the range of 18,000–20,000 N, which means about a 35–40% reduction compared to the maximum force in extreme designs. Thus, the research goal of maximizing absorbed energy while minimizing maximum reaction force has been achieved quantitatively.

This study demonstrates that the RSM approach based on the Central Composite Design (CCD) is effective for modeling and optimizing the axial impact behavior of aluminum alloy tubes. The results obtained not only enrich the theoretical understanding of the energy-absorption mechanism and reaction force in thin-walled tube structures, but also provide a strong numerical basis for the design of energy-absorbing elements in numerical-simulation-based crashworthiness applications.

REFERENCES

Adachi, T., A. Tomiyama, W. Araki, and A. Yamaji. 2018. "Energy absorption of a thin-walled cylinder with ribs subjected to axial impact," *Int. J. Impact Eng.*, vol. 35, no. 2, pp. 65–79. doi: 10.1016/j.ijimpeng.2006.11.005.

Ahmad, Z. 2009. "Impact and Energy Absorption of Empty and Foam-filled Conical Tubes," *Proceedings of COBEM.*

Aktaş, C., E. Acar, M. A. Güler, and M. Altın. 2022. "An investigation of the crashworthiness performance and optimization of tetra-chiral and reentrant crash boxes," *Mechanics Based Design of Structures and Machines*, 2022, doi: 10.1080/15397734.2022.2075382.

Alghamdi, A. A. A.. 2001. "Collapsible impact energy absorbers: An overview," *Thin-Walled Structures*, vol. 39, no. 2, pp. 189–213, doi: 10.1016/S0263-8231(00)00048-3.

B. G and A. K. 2019. "Numerical investigation of honeycomb filled crash box for the effect of honeycomb's physical parameters on crashworthiness constants," *International Journal of Crashworthiness*, vol. 24, no. 2, pp. 184–198, Mar. 2019, doi:10.1080/13588265.2018.1424298.

Ferdynus, M., P. Rozylo, and M. Rogala. 2020. "Energy absorption capability of thin-walled prismatic aluminum tubes with spherical indentations," *Materials*, vol. 13, no. 19, pp. 1–19. doi: 10.3390/ma13194304.

Galib, D. Al. and A. Limam. 2004."Experimental and numerical investigation of static and dynamic axial crushing of circular aluminum tubes," *Thin-Walled Structures*, vol. 42, no. 8, pp. 1103–1137. doi: 10.1016/j.tws.2004.03.001.

Ghamarian, A. and M. Tahaye Abadi. 2011. "Axial crushing analysis of end-capped circular tubes," *Thin-Walled Structures*, vol. 49, no. 6, pp. 743–752, 2011, doi:10.1016/j.tws.2011.01.006.

Günaydın, K., O. Gülcan, and H. S. Türkmen. 2023. "Experimental and numerical crushing performance of crash boxes filled with re-entrant and anti-tetrachiral auxetic structures," *International Journal of Crashworthiness*, vol. 28, no. 5, pp. 649–663. doi:10.1080/13588265.2022.2115962.

Guoxing Lu and Tongxi Yu. 2013. *Energy absorption of structures and materials*. Cambridge: CRC Press LLC.

Hardi, W. 2022. *Analisis elemen hingga untuk uji tabrakan mobil dengan ansys ls dyna*. Bandung: Media Sains Indonesia.

Idesman, A. V. and S. P. Mates. 2014. "Accurate finite element simulation and experimental study of elastic wave propagation in a long cylinder under impact loading," *Int. J. Impact Eng.*, vol. 71, pp. 1–16. doi: 10.1016/j.ijimpeng.2014.04.002.

Jones, N. 1989. *Structural Impact*. Cambridge,

Mat, F., K. A. Ismail, S. Yaacob, and O. Inayatullah. 2012. "Impact response of thin-walled tubes: A prospective review," *Applied Mechanics and Materials*, vol. 165, no. March 2014, pp. 130–134, doi: 10.4028/www.scientific.net/AMM.165.130.

Mathiazhagan, N., S. Palaniyappan, and N. kumar Sivakumar. 2022. "Effect of fused filament fabrication parameters on crashworthiness studies of hydroxyapatite particle reinforced PLA composite thin-walled tubes," *J. Mech. Behav. Biomed. Mater.*, vol. 138, Feb. 2023, doi: 10.1016/j.jmbbm.2022.105611.

Nagel, G. 2025. "(TEZ) Impact and Energy Absorption of Straight and Tapered Rectangular Tubes," no. February, p. 324.

Toksoy, A. K. and M. Güden. 2011. "The optimisation of the energy absorption of partially Al foam-filled commercial 1050H14 and 6061T4 Al crash boxes," *International Journal* *of Crashworthiness*, vol. 16, no. 1, pp. 97–109, Feb. 2011, doi: 10.1080/13588265.2010.514774.

Wei, Z. G., J. L. Yu, and R. C. Batra. 2016. “Dynamic buckling of thin cylindrical shells under axial impact,” *Int. J. Impact Eng.*, vol. 32, no. 1–4, pp. 575–592. doi:10.1016/j.ijimpeng.2005.07.008.

Witteman, W. J.. 1999. *Improved Vehicle Crashworthiness Design by Control of the Energy Absorption for Different Collision Situations*. doi: 10.6100/IR518429.

Xu, F., K. Yu, L. Hua, and X. Niu. 2022. “Crashworthiness design of crash box filled with negative Poisson’s ratio based on horn structure,” *Mechanics of Advanced Materials and Structures*, vol. 29, no. 27, pp. 6403–6420. doi: 10.1080/15376494.2021.1978594.

Zarei, H. R. and M. Kröger. 2007. “Crashworthiness optimization of empty and filled aluminum crash boxes,” *International Journal of Crashworthiness*, vol. 12, no. 3, pp. 255–264. doi: 10.1080/13588260701441159.