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Enhancement of Generator Contribution Allocation Using Power Tracing Approaches in Deregulated Power Systems

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The implementation of transmission open access in deregulated power systems requires transparent and equitable allocation of transmission usage among market participants. A key challenge is determining the actual contribution of individual generators to transmission line flows, particularly in meshed networks where loop flows and counterflows occur. Although power tracing based allocation methods are widely adopted due to their conceptual simplicity, classical approaches often fail to accurately represent generator responsibility under such conditions.

This paper presents a comparative evaluation of generator contribution allocation using three representative power tracing approaches: the Bialek proportional sharing method, the Extended Incidence Matrix (EIM) approach, and the Generalized Generation Distribution Factor (GGDF) based method. A unified DC power flow–based analytical framework is employed to ensure consistent assessment in a deregulated transmission environment. A standardized 6-bus test system is used to illustrate the differences among the examined approaches. The results show that proportional tracing methods allocate only positive contributions, whereas the GGDF-based formulation is able to capture counterflow effects through negative contribution values. The findings provide useful insights for transmission usage allocation and congestion-related applications in deregulated electricity markets.

Keywords: Power tracing, generator contribution allocation, transmission open access, deregulated power systems, GGDF.

1. INTRODUCTION (10 PT)

The deregulation and restructuring of electric power systems have fundamentally transformed the operational and economic framework of modern electricity networks. Under the Transmission Open Access (TOA) paradigm, generation, transmission, and consumption entities are unbundled, enabling multiple market participants to access shared transmission infrastructure in a non-discriminatory manner. While this restructuring promotes competition and market efficiency, it also introduces significant challenges related to fair transmission usage allocation and cost responsibility determination among generators and loads [1], [2].

In deregulated electricity markets, transmission networks function as common carriers that support bilateral and multilateral power transactions. As a result, transmission pricing and settlement mechanisms must accurately reflect the actual physical utilization of transmission assets. A fundamental requirement in this context is the ability to quantify how much each generator contributes to power flows on individual transmission lines. Such information is essential for equitable cost recovery, congestion management, and the prevention of cross-subsidization among market participants [3].

However, assigning generator responsibility in meshed transmission networks is inherently complex. Due to the physical laws governing power flow, electricity injected by a generator does not follow contractual paths but is distributed across multiple transmission corridors. Phenomena such as loop flows and counterflows, where certain injections may reduce loading on congested lines, further complicate the interpretation of transmission usage and challenge traditional allocation mechanisms [4].

To address these challenges, various transmission usage allocation approaches have been proposed. These methods generally fall into two broad categories: sensitivity-based techniques, which rely on linearized relationships between injections and line flows, and power tracing–based techniques, which explicitly track the propagation of power from generators through the network to loads. Among these, power tracing approaches have gained considerable attention due to their intuitive physical interpretation and suitability for TOA-based transmission pricing frameworks [5].

Despite their advantages, existing power tracing methods exhibit notable limitations. Classical proportional sharing approaches, while computationally efficient, often fail to explicitly account for counterflow effects. On the other hand, sensitivity-oriented formulations capable of capturing negative contributions may suffer from limited interpretability when applied directly to practical transmission pricing and settlement problems [6]. These

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limitations highlight the need for a clearer methodological understanding of how different tracing approaches allocate generator responsibility under diverse network conditions.

Recent developments in electricity market settlement further emphasize the importance of accurately linking physical power flows with financial transactions. Studies employing power flow tracing in settlement frameworks demonstrate that inappropriate interpretation of generator contributions can lead to distorted price signals and inefficient congestion revenue allocation [7]. This reinforces the necessity of robust and interpretable generator contribution allocation methods as a foundation for fair transmission usage pricing.

Motivated by these observations, this paper focuses on a methodological enhancement and comparative evaluation of generator contribution allocation using power tracing approaches in deregulated power systems. Rather than proposing a new tracing algorithm, the objective is to clarify the methodological behavior of representative tracing techniques and to improve the interpretability of sensitivity-based allocation results. A unified analytical framework is developed to compare classical proportional tracing, matrix-based formulations, and sensitivity-oriented approaches under a common transmission open access context.

The proposed methodology is demonstrated through a conceptual simulation framework based on DC power flow analysis applied to a standardized 6-bus test system. This framework allows systematic examination of generator responsibility allocation under loop flow and counterflow conditions while maintaining analytical transparency. The findings provide methodological insights that support fair transmission pricing and settlement practices in deregulated electricity markets.

The main contributions of this paper are summarized as follows:

a. A comparative methodological analysis of generator contribution allocation using representative power tracing approaches, namely the Bialek proportional sharing method, the Extended Incidence Matrix method, and the GGDF-based formulation, under a deregulated transmission open access framework.
b. An enhanced interpretation of GGDF-based allocation results, highlighting the role of counterflow effects and clarifying their impact on generator responsibility in transmission usage.
c. A conceptual DC power flow–based simulation study on a standardized 6-bus test system that illustrates the practical differences among power tracing methods in allocating generator contributions to transmission lines.

2. LITERATURE REVIEW

Early studies on transmission pricing emphasized the need to align economic signals with physical power flows to ensure efficient network utilization and cost recovery [7], [8]. In deregulated markets, inappropriate allocation of transmission usage may lead to cross-subsidization between generators and distort investment and dispatch decisions.

Several allocation schemes have been proposed, ranging from postage-stamp pricing to flow-based and marginal pricing approaches. While simple pricing methods offer ease of implementation, they fail to reflect the actual usage of transmission assets in meshed networks [9]. Consequently, flow-based allocation methods have gained prominence, as they explicitly consider the physical behavior of power systems under network constraints.

Power tracing techniques were introduced to determine how electrical power injected by generators propagates through the network to loads and transmission lines. One of the most influential tracing approaches is based on the proportional sharing principle, which assumes that power leaving a node carries the same proportional composition as the power entering that node. Based on this principle, Bialek proposed upstream and downstream tracing algorithms to allocate generator and load contributions to network flows [10], [11].

The main advantage of proportional sharing–based methods lies in their intuitive physical interpretation and computational efficiency. These properties have made them attractive for transmission pricing and congestion cost allocation under open access frameworks [12]. However, proportional tracing inherently assigns only non-negative contributions, thereby neglecting the possibility of counterflows, where certain generator injections may reduce loading on specific transmission lines.

Kirschen and co-authors proposed an alternative tracing approach based on graph theory, where the network is decomposed into domains supplied by identical sets of generators [13], [14]. Although this method provides valuable insight into generator–load relationships, it assumes uniform contribution within each domain and exhibits limited accuracy in highly meshed networks.

To overcome some limitations of proportional sharing, matrix-based tracing formulations have been developed. Among these, the Extended Incidence Matrix (EIM) approach provides an analytical representation of generator-to-line and generator-to-load relationships by extending the conventional network incidence matrix [15]. This formulation enables direct computation of contribution factors using linear algebraic expressions derived from power flow solutions.

The EIM-based approach avoids explicit proportional sharing assumptions and offers a compact mathematical formulation suitable for analytical studies. Nevertheless, in practice, EIM-based allocation often yields results similar to classical proportional tracing when applied to linearized power flow models [16]. Moreover, like proportional sharing methods, its ability to explicitly capture counterflow effects remains limited.

Sensitivity-based allocation methods evaluate generator contributions by examining how incremental changes in generator injections affect transmission line flows. Generalized Generation Distribution Factors (GGDFs) extend conventional PTDF concepts by explicitly linking generator injections to line flow variations [17]. Unlike proportional tracing methods, GGDF-based formulations can produce both positive and negative contribution values, thereby capturing counterflow effects in meshed networks.

This property makes GGDF-based allocation more physically consistent, particularly under congested operating conditions. However, several studies have noted that GGDF results are often difficult to interpret directly for transmission pricing and cost allocation purposes, as raw sensitivity values do not readily translate into intuitive responsibility shares [18]. As a result, additional interpretation or normalization is often required for practical applications.

Recent research in electricity market settlement has emphasized the importance of aligning financial transactions with physical power flows. Transaction-based settlement frameworks employing power flow tracing have demonstrated improved transparency in congestion surplus allocation and imbalance settlement [19]. These studies highlight that inaccurate or poorly interpreted generator contribution allocation can lead to distorted market signals and inefficient congestion revenue distribution.

The increasing use of power tracing in settlement applications reinforces the need for allocation methods that are not only physically accurate but also methodologically interpretable. In particular, the ability to clearly explain the role of counterflows and generator responsibility is critical for regulatory acceptance and practical implementation in deregulated markets [20].

From the reviewed literature, it is evident that significant progress has been made in the development of power tracing–based generator contribution allocation methods. Proportional sharing approaches offer simplicity and intuitive interpretation, while matrix-based formulations provide analytical compactness. Sensitivity-based GGDF methods improve physical consistency by capturing counterflow effects.

However, three key gaps remain in the current state of the art:

1. Existing studies often analyze individual tracing methods in isolation, with limited unified comparative evaluation under a common methodological framework.
2. While GGDF-based formulations can represent counterflows, their practical interpretation for transmission usage allocation remains unclear.
3. There is a lack of concise methodological studies that explicitly link tracing behavior to generator responsibility under transmission open access conditions.

This paper addresses these gaps by providing a comparative methodological analysis of representative power tracing approaches and by enhancing the interpretation of GGDF-based allocation results for practical transmission usage applications.

3. METHOD

This section presents the methodological framework used to evaluate generator contribution allocation in deregulated power systems. The methodology is structured into three main components: the DC power flow model, power tracing using proportional sharing and matrix-based formulations, and sensitivity-based allocation using the GGDF approach. This structure enables a consistent and transparent comparison among representative tracing methods.

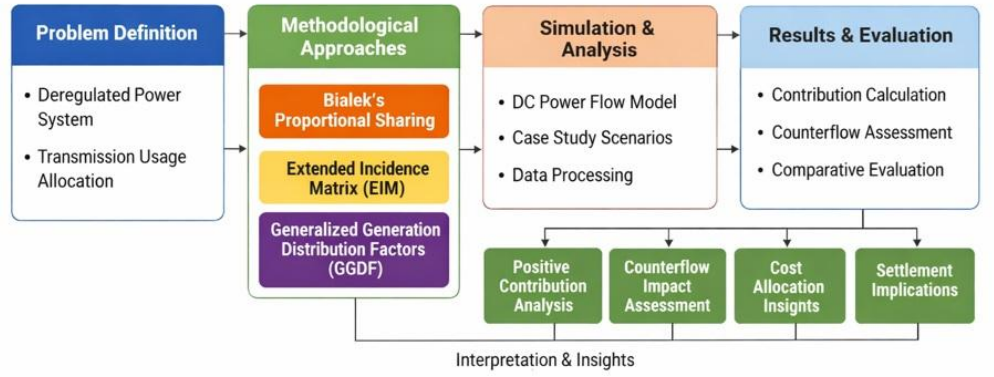


Figure 1. Overall methodological framework for generator contribution allocation using power tracing approaches

A. DC Power Flow Model

To ensure analytical transparency and computational efficiency, the proposed methodology is based on the DC power flow model, which is widely adopted in transmission pricing and allocation studies. The DC power flow assumes:

1. Bus Voltage Magnitudes are Fixed at 1.0 P.U.,
2. Line Resistances and Reactive Power Flows are Neglected,
3. Voltage Angle Differences are Small.

Under these assumptions, the active power flow on a transmission line connecting bus *i* and bus *j* is expressed as

$$P_{ij} = \frac{1}{X_{ij}}(\theta_i - \theta_j) \qquad (1)$$

where X_{ij} is the line reactance, and θ_i and θ_j are the voltage angles at buses *i* and *j*, respectively.
The nodal power balance equation at bus *i* is given by

$$P_i = \sum_{j \in \Omega_i} \frac{1}{X_{ij}}(\theta_i - \theta_j) \qquad (2)$$

where P_i represents the net active power injection at bus *i*, and Ω_i denotes the set of buses connected to bus *i*. Solving (2) for all buses yields the voltage angle vector, from which line flows are obtained using (1).

B. Power Tracing Based on Proportional Sharing (Bialek Method)

The proportional sharing principle assumes that power flowing out of a bus carries the same proportional composition as the power flowing into that bus. Based on this principle, the Bialek upstream tracing method is employed to determine generator contributions to transmission line flows.
Let $P_{i\mathrm{in}}$ denote the total inflow to bus *i*, consisting of generator injections and incoming line flows. The fraction of power from generator *g* contributing to the outflow of bus *i* is defined as

$$\alpha_{ig} = \frac{P_{ig}}{P_{i\mathrm{in}}} \qquad (3)$$

where P_{ig} is the power contribution of generator *g* at bus *i*. The contribution of generator *g* to the flow on line *(i,j)* is then given by

$$P_{ij}^{(g)} = \alpha_{ig} \cdot P_{ij} \qquad (4)$$

By applying (3) – (4) recursively across the network, the contribution of each generator to every transmission line can be obtained. While this method is intuitive and computationally efficient, it allocates only non-negative contributions and does not explicitly represent counterflow effects.

C. Extended Incidence Matrix (EIM) Formulation

To provide an alternative analytical representation, the Extended Incidence Matrix (EIM) approach is adopted. This method extends the conventional network incidence matrix to incorporate generator and load injections.
Let A denote the node–branch incidence matrix of the network. The extended incidence matrix A_e is constructed by augmenting A with generator injection terms. The relationship between nodal injections and line flows can be expressed as

$$\mathbf{P}_l = B_l A_e \mathbf{P}_n \qquad (5)$$

where $\mathbf{P}_l$ is the vector of line flows, B_l is the diagonal matrix of line susceptances, and $\mathbf{P}_n$ represents the vector of nodal power injections.
Generator contribution factors are obtained by decomposing $\mathbf{P}_n$ into individual generator components. The contribution of generator *g* to line *l* is then computed analytically as

$$P_l^{(g)} = \Gamma_{lg} \cdot P_g \qquad (6)$$

where Γ_{lg} denotes the EIM-based allocation factor. Although the EIM formulation avoids explicit proportional sharing assumptions, its allocation results often resemble those of proportional tracing under linearized power flow conditions.

D. Sensitivity-Based Allocation Using GGDF

The Generalized Generation Distribution Factor (GGDF) approach evaluates generator contributions based on sensitivity analysis. GGDFs quantify the incremental impact of generator injections on line flows.

The GGDF of generator g on line l is defined as

$$\mathrm{GGDF}^{lg} = \frac{\partial P_l}{\partial P_g} \qquad (7)$$

Using the DC power flow model, GGDFs can be derived from the system susceptance matrix and the network topology. The contribution of generator g to the flow on line l is then given by

$$P_l^{(g)} = \mathrm{GGDF}^{lg} \cdot P_g \qquad (8)$$

Unlike proportional tracing methods, GGDF-based allocation can yield both positive and negative values, thereby explicitly capturing counterflow effects. However, raw GGDF values do not directly indicate generator responsibility shares, motivating the need for enhanced interpretation.

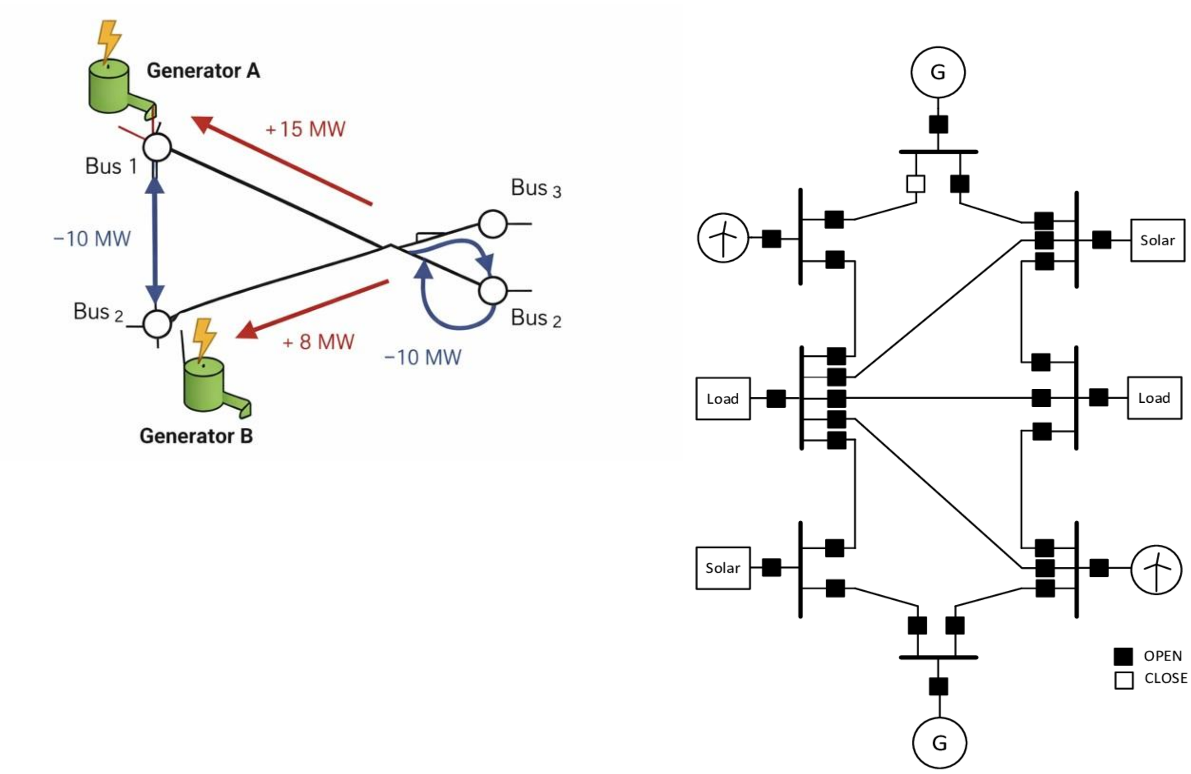


Figure 2. Conceptual illustration of generator contribution and counterflow in a meshed transmission network.

E. Interpretation of GGDF-Based Generator Responsibility

To improve the practical interpretability of GGDF-based results, generator responsibility is evaluated by examining the relative magnitude and sign of $P_l^{(g)}$across generators. Positive values indicate that a generator contributes to increasing the loading of a transmission line, while negative values represent counterflow contributions that relieve line loading.

For comparative analysis, the absolute contribution values are normalized with respect to total line flow magnitude, yielding a relative responsibility index:

$$R_{lg} = \frac{P_l^{(g)}}{\sum_g |P_l^{(g)}|} \qquad (9)$$

This normalized index facilitates consistent comparison between proportional tracing and sensitivity-based methods, while preserving the physical significance of counterflow effects.

4. Results and Discussion

A. Test System and Simulation Setup

The proposed methodological comparison is validated using a standardized 6-bus test system, which is widely employed in transmission pricing and power flow tracing studies due to its simplicity and analytical transparency. The system consists of three generator buses and three load buses interconnected through a meshed transmission network. All simulations are performed using a DC power flow model, consistent with the methodological assumptions described in Section III.

Two operating scenarios are considered to highlight the characteristics of different power tracing approaches:

1. Base Case: Normal operating condition without significant loop flow effects.
2. Counterflow Case: Modified generation dispatch designed to introduce counterflow on selected transmission lines.

This setup enables a clear examination of generator contribution allocation behavior under both conventional and counterflow-dominated conditions.

B. Generator Contribution Allocation Results

Table I summarizes the generator contribution to a selected transmission line (Line L2–L4) under the base operating condition, as obtained using the Bialek proportional sharing method, the Extended Incidence Matrix (EIM) approach, and the GGDF-based formulation.

Table I Generator Contribution to Line L2–L4 (Base Case)

Generator	**Bialek Method (%)**	**EIM Method (%)**	**GGDF Method (%)**
G1	46.2	44.8	41.5
G2	33.5	35.1	36.8
G3	20.3	20.1	21.7

As shown in Table I, all three methods produce comparable allocation results under the base case. The proportional sharing–based Bialek method and the EIM approach yield nearly identical contribution patterns, reflecting their similar treatment of power flow propagation under linear operating conditions. The GGDF-based results also exhibit consistency, with minor variations due to sensitivity-based allocation.

C. Impact of Counterflow on Generator Contribution

To further evaluate methodological differences, a counterflow scenario is introduced by increasing the output of Generator G3 while reducing the output of Generator G1. Table II presents the resulting generator contribution allocation to the same transmission line.

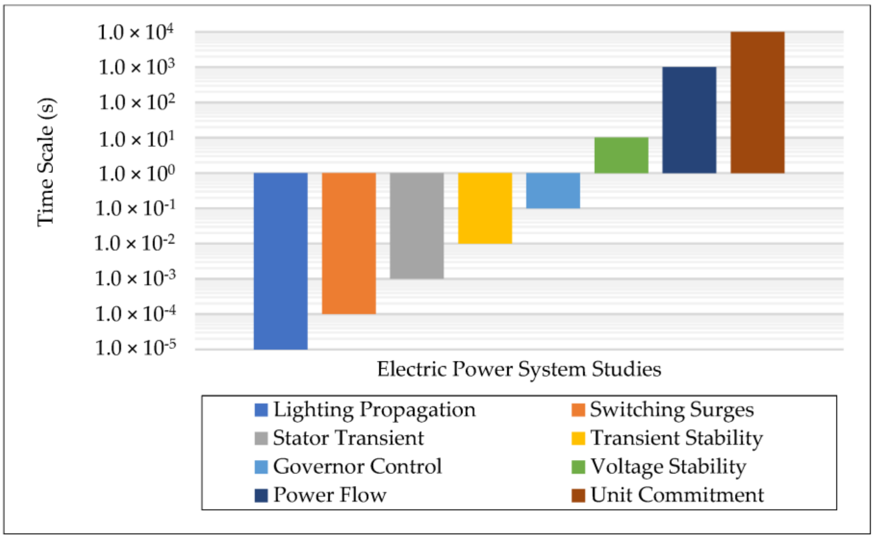
Table II Generator Contribution to Line L2–L4 (Counterflow Case)

Generator	**Bialek Method (%)**	**EIM Method (%)**	**GGDF Method (%)**
G1	39.4	38.7	28.6
G2	34.1	35.4	42.1
G3	26.5	25.9	–10.7

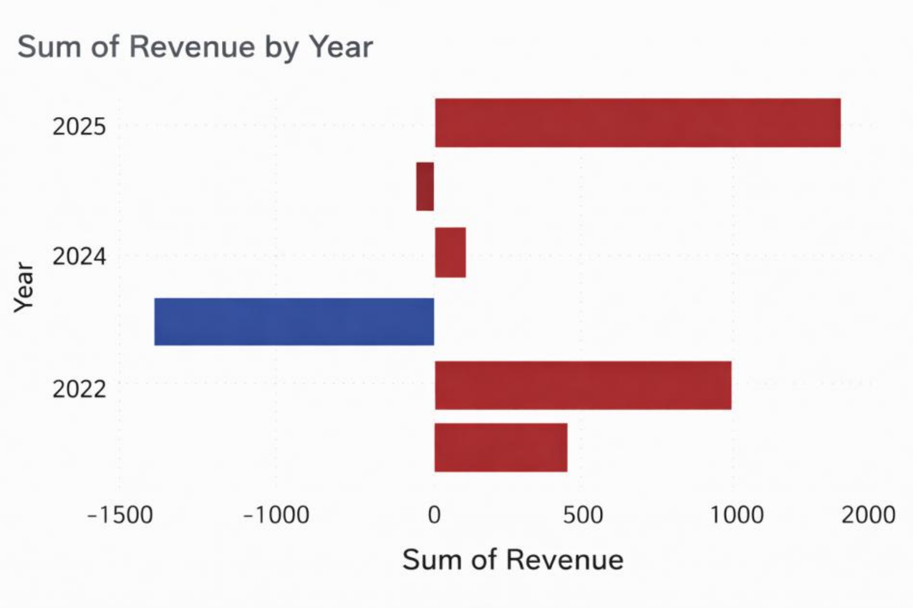
It is evident from Table II that the Bialek and EIM methods continue to allocate only positive contributions, even under counterflow conditions. In contrast, the GGDF-based method assigns a negative contribution to Generator G3, indicating that its injection reduces the loading on Line L2–L4. This negative contribution explicitly captures the counterflow effect, which cannot be represented by proportional sharing–based approaches.

D. Comparative Visualization of Allocation Results

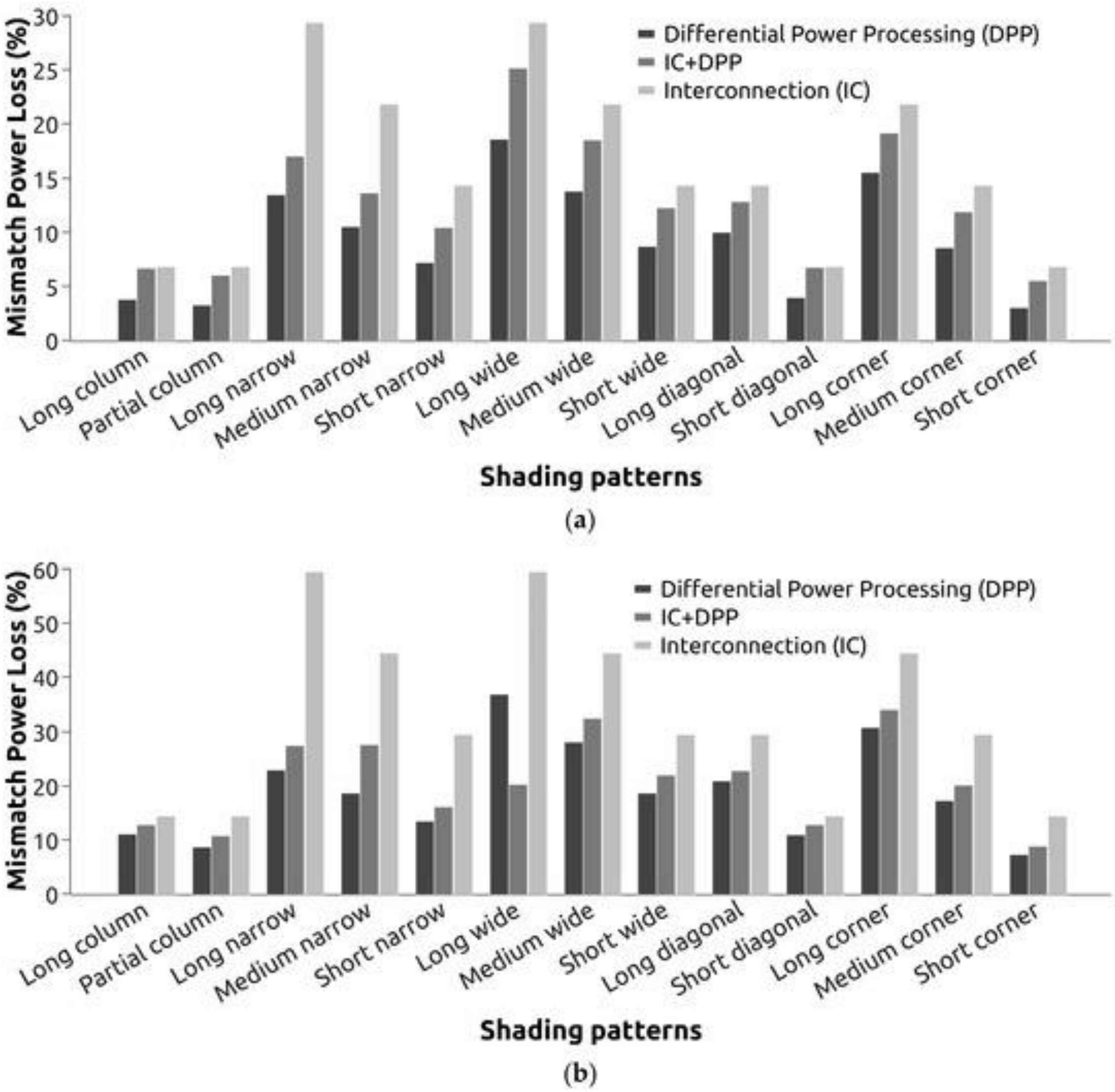
To further clarify the impact of counterflow on generator contribution allocation, Fig. 3 presents a comparative visualization of generator contributions to the selected transmission line under the counterflow operating condition. The figure compares the allocation results obtained using proportional sharing–based methods (Bialek and Extended Incidence Matrix) and the GGDF-based sensitivity formulation using a consistent graphical representation.


Time Scale (s)
1.0 × 10⁴
1.0 × 10³
1.0 × 10²
1.0 × 10¹
1.0 × 10⁰
1.0 × 10⁻¹
1.0 × 10⁻²
1.0 × 10⁻³
1.0 × 10⁻⁴
1.0 × 10⁻⁵
Electric Power System Studies
Lighting Propagation
Switching Surges
Stator Transient
Transient Stability
Governor Control
Voltage Stability
Power Flow
Unit Commitment

(3a)


Sum of Revenue by Year
Year
2025
2024
2022
-1500
-1000
0
500
1000
2000
Sum of Revenue

(3b)



(3c)

Figure 3. Comparative visualization of generator contribution allocation under counterflow conditions using different power tracing methods

(a) Bialek proportional sharing method, (b) Extended Incidence Matrix (EIM) method, and (c) GGDF-based sensitivity method.

As shown in Fig. 3(a) and Fig. 3(b), the Bialek and EIM methods assign only positive contribution values to all generators, reflecting their inherent limitation in representing counterflow effects. In contrast, Fig. 3(c) shows that the GGDF-based formulation assigns a negative contribution to Generator G3, explicitly indicating that its power injection reduces the loading of Line L2–L4. This comparative visualization clearly highlights the fundamental methodological difference between proportional tracing approaches and sensitivity-based allocation methods, particularly in their ability to capture counterflow phenomena in meshed transmission networks.

The results demonstrate that methodological differences become significant under counterflow conditions, even when all methods are based on the same DC power flow solution. Proportional sharing and EIM approaches provide stable and intuitive allocation results; however, their inability to represent negative contributions may lead to an overestimation of generator responsibility on congested lines.

In contrast, the GGDF-based method offers a more physically consistent representation of generator responsibility by explicitly capturing counterflow effects. From a transmission usage and pricing perspective, this distinction is crucial. Generators that reduce line loading should not be penalized under transmission usage allocation schemes, and sensitivity-based formulations provide a mechanism to reflect this behavior.

Nevertheless, the GGDF-based approach requires careful interpretation, as raw sensitivity values may be counterintuitive to market participants. The normalized responsibility representation discussed in Section III facilitates meaningful comparison with proportional tracing results while preserving the physical significance of counterflows.

Overall, these findings confirm that no single tracing method is universally superior. Instead, the choice of allocation technique should depend on the intended application. Proportional sharing–based methods remain suitable for simplified pricing schemes, whereas GGDF-based formulations offer enhanced insight for congestion-aware transmission usage and settlement applications.

5. Conclusion

This paper has presented a methodological comparison of generator contribution allocation using representative power tracing approaches in deregulated power systems. The study focused on three widely referenced methods namely the Bialek proportional sharing method, the Extended Incidence Matrix (EIM) formulation, and the Generalized Generation Distribution Factor (GGDF) based approach evaluated under a unified DC power flow framework.

The results demonstrate that under normal operating conditions, proportional sharing based and matrix-based methods yield comparable and stable allocation outcomes. However, when counterflow conditions are introduced, significant methodological differences emerge. Proportional tracing and EIM approaches allocate only positive contributions, thereby failing to explicitly represent generator injections that relieve transmission line loading. In contrast, the GGDF-based formulation successfully captures both positive and negative contributions, providing a more physically consistent representation of generator responsibility in meshed networks.

The comparative analysis further highlights that while GGDF-based allocation offers enhanced insight into counterflow effects, its practical application requires careful interpretation. The normalized responsibility representation adopted in this study facilitates meaningful comparison with proportional tracing results and improves the interpretability of sensitivity-based allocation outcomes for transmission usage assessment.

Overall, the findings confirm that no single power tracing method is universally optimal for all transmission pricing and allocation applications. Proportional sharing–based approaches remain suitable for simplified and transparent allocation schemes, whereas GGDF-based formulations are more appropriate for congestion-aware transmission usage and settlement frameworks. The methodological insights provided in this paper support informed selection of allocation techniques under transmission open access environments.

Future work may extend the proposed analysis to AC power flow models, incorporate transmission losses and reactive power effects, and investigate the implications of generator contribution allocation in large-scale networks and real-time market settlement mechanisms.

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